

## HEAT TRANSFER OF A DISK ROTATING IN A CASING

(NASA-TT-F-15199) V. M. Kapinos  
DISK ROTATING IN A CASING (Scientific  
Translation Service) 18 p HC \$3.00  
17

CSCL 20M

N74-12572

H2/33

Unclass  
23325

Translation of: "Teploobmen diska,  
vrashchayushchevosya v kozhukhe," Izvestiya  
vysshikh uchebnykh zavedeniy, seriya, "Aviats-  
ionnaya Tekhnika," Vol. 8, No. 2, 1965, pp.  
76 — 86.



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
WASHINGTON, D. C. 20546  
DECEMBER 1973

# HEAT TRANSFER OF A DISK ROTATING IN A CASING

V. M. Kapinos

ABSTRACT. Discussion of the heat transfer between a heated rotating disk and a cooling fluid flowing radially from the center to the periphery in the gap between the disk and the casing of a gas turbine (with cooled rotors). The heat transfer coefficient is determined making use of the Reynolds analogy.

## Notation

$v, v_r, v_z$  — peripheral, radial, and axial components of velocity

/76\*

$r, R$  — radial coordinate and maximum radius of disk

$r_0$  — radius at which cooling fluid enters gap

$r_1$  — radius at which boundary layers of disk and casing join

$x = \frac{r}{r_0}, X = \frac{R}{r_0}$  — relative radii

$z$  — coordinate measured along normal to disk face

$\delta, \delta_c$  — thicknesses of boundary layer at disk and casing

$\omega$  — angular velocity of disk

$Q$  — volume flow rate of cooling fluid

$s$  — gap width

\*Numbers in the margin indicate pagination of original foreign text.

- $\nu$  — kinematic modulus of viscosity;
- $\alpha_n, \bar{\alpha}$  — local and average heat transfer coefficients
- $t_d$  — disk temperature
- $t_0$  — initial temperature of cooling fluid
- $\tau$  — flow twist equal to  $\frac{V_\varphi}{r\omega}$ .

Heat transfer between a heated rotating disk and a cooling fluid flowing radially from the center to the periphery in the gap between the disk and the casing is found in gas turbines with cooled rotors.

We consider flow in the gap without initial twist. In this case, the flow core remains untwisted right up to the section at which the boundary layers at the disk and casing join. Actually, the equation of motion of the nonviscous flow core between the boundary layers projected onto the peripheral direction has the form:

$$V_r \frac{\partial V_\varphi}{\partial r} + \frac{V_r V_\varphi}{r} + V_z \frac{\partial V_\varphi}{\partial z} = 0. \quad (1)$$

If the axial velocity  $V_z = 0$ , integration of Equation (1) gives:

$$\zeta = \frac{V_\varphi}{r\omega} = \frac{C}{r^2}.$$

The constant of integration  $C = 0$ , since at the input to the gap with  $r = r_0$  by definition  $\zeta = 0$ . Consequently, the peripheral velocity in the flow core is also equal to zero up to the section where viscous flow in the boundary layers extends over the whole width of the gap. Such a flow characteristic is verified by experiments performed in [1].



Absence of twist of the external flow (relative to the boundary layer) permits one to assume the peripheral velocity component profile in the turbulent boundary layer at the disk with radial blowing out, just as for the freely rotating disk [2]:

$$V_\theta = r\omega \left[ 1 - \left( \frac{z}{\delta} \right)^{1/2} \right] \quad (2)$$

Experiments carried out by V. S. Sedach showed that for high flow rates, the radial velocity profile in the gap between disks and casing is convex, close to the velocity profile in a tube. In the case of low flow rates, the form of the radial velocity profile is expected to be the same as the flow around a freely rotating disk or a disk rotating in a casing without flow [3, 4]. Consequently, the radial velocity distribution in the boundary layer takes the form:

$$V_r = \left( \frac{z}{\delta} \right)^{1/2} \left[ \cos \left( 1 - \frac{z}{\delta} \right) + V_{r0} \right], \quad V_{r0} = \frac{Q}{2\pi r \delta} \quad (3)$$

A similar binomial representation for the radial velocity with flow was proposed by A. L. Kyznetsov [5]. At  $V_{r0} \rightarrow 0$  and for the value of the parameter  $z = 0.16$ , the profile (3) coincides with the profile obtained by T. Karman for a freely rotating disk. If the flow rate of the cooking fluid is large, such that  $\cos \ll V_{r0}$ , then the radial velocity distribution is analogous to the velocity distribution in a tube. This case of flow is considered in [6].

The peripheral and radial velocity component profiles (2) and (3) satisfy the following boundary conditions: for

$$z=0 \quad V_\theta = r\omega, \quad V_r = 0; \quad \text{for } z=\delta \quad V_\theta = 0, \quad V_r = V_{r0}.$$

We substitute (2), (3) into the equation for the angular momentum of the boundary layer at the disk:

$$\rho \frac{d}{dr} \left( r^2 \int_0^{\delta} V_r V_z dz \right) = - r^2 \tau_r,$$

obtaining the flow stress according to the Blasius equation equal to:

$$\tau_r = - 0.0225 \frac{V_0^{7/4} \nu^{1/4}}{x^{1/4}} \cos \beta, \quad (4)$$

Here the total velocity vector of flow around the disk is:

$$V_0 = \omega r \left[ 1 + \left( z + \frac{1}{K_p x^2} \right)^2 \right]^{0.5}, \quad K_p = 2\pi r_0^2 \frac{\omega s}{Q}, \quad (5)$$

and the angle between this vector and the peripheral direction:

$$\beta = \arccos \left[ 1 + \left( z + \frac{1}{K_p x^2} \right)^2 \right]^{-0.5}.$$

Then to determine the boundary layer thickness, we obtain the equation:

$$\begin{aligned} \frac{d}{dx} \left[ \delta \left( 0.0681 z x^4 + 0.0972 \frac{x^2}{K_p} \right) \right] = \\ = 0.0225 x^{15/4} r_0^{3/4} \left[ 1 + \left( z + \frac{1}{K_p x^2} \right)^2 \right]^{3/8} \left( \frac{\nu}{\omega \delta} \right)^{1/4}. \end{aligned} \quad (6)$$

Integrating (6) with the initial condition  $x = 1, \delta/r = 0$  gives: 78

$$\frac{\delta}{r} = 0.493 \frac{K_p a^{0.5}}{\text{Re}_a^{0.2} x^{2.6} (z K_p x^2 + 1.428)}, \quad (7)$$

$$a(K_p, x) = \int_1^x x^{4.75} \left[ 1 + \left( z + \frac{1}{K_p x^2} \right)^2 \right]^{0.975} \left( z + \frac{1.428}{K_p x^2} \right)^{0.25} dx. \quad (8)$$

Combining Expressions (4) and (7), we find the dimensionless flow stress and the coefficient of drag torque:

$$\frac{\tau_p}{\rho(r\omega)^2} = 0.0268 \frac{x^{1.15} \left[ 1 + \left( 1 + \frac{1}{K_p x^2} \right)^2 \right]^{0.375} \left( 1 + \frac{1.428}{K_p x^2} \right)^{0.25}}{Re_n^{0.2} a^{0.2}}, \quad (9)$$

$$C_n = \frac{2M}{\rho \omega^2 R^4}, \quad M = 2\pi \int_0^R \tau_p r^2 dr, \quad C_n = 0.422 \frac{a^{0.8}}{Re_n^{0.2} X^{4.6}}, \quad (10)$$

For  $X \rightarrow \infty$  and  $K_p \rightarrow \infty$  the coefficient of drag torque:

$$C_n = \frac{0.073}{Re_n^{0.2}},$$

which coincides with the solution of T. Karman [2].

We use the Reynold's analogy to obtain the heat transfer coefficient. It is shown in [7] that, for the Prandtl number  $Pr = 1$  and a quadratic distribution of temperature head with radius of a freely rotating disk, a similarity of temperature and peripheral velocity component profiles occurs. The latter will also be similar in the case being considered, if the temperature head is reckoned from the initial temperature of the cooling fluid [6]. It follows from their similarity:

$$Nu_n = \frac{\tau_p}{\rho(r\omega)^2} Re_n, \quad Nu_n = \frac{\tau_n r}{\lambda}. \quad (11)$$

Substituting (9) into (11), we have:

$$Nu_n = 0.0268 Re_n^{0.8} A_n(K_p, x),$$

$$A_n(K_p, x) = \frac{\left[ 1 + \left( 1 + \frac{1}{K_p x^2} \right)^2 \right]^{0.375} \left( 1 + \frac{1.428}{K_p x^2} \right)^{0.25} x^{1.15}}{a^{0.2}}. \quad (12)$$



The average value of the Nusselt number will be, by definition:

$$Nu = R^2 \frac{\int_0^R Nu_n (t_d - t_0) dr}{\int_0^R (t_d - t_0) r dr}, \quad Nu = \frac{qR}{\lambda} \quad (13)$$

Noting that  $t_d - t_0 = kr^2$  ( $k$  — arbitrary number) and inserting (12) into (13), we find:

$$Nu = 0.134 Re^{0.8} A(K_0, X), \quad A(K_0, X) = \frac{a^{0.8}}{X^{0.8}(X^4 - 1)} \quad (14)$$

By using the particular solutions of (12) and (14) satisfying a quadratic law for the variation of excess disk temperature, one can determine the heat transfer coefficient for an arbitrary temperature distribution  $t_d(r)$ . The calculation procedure is given in detail in [6, 8]. The final relationships have the form: /79

$$Nu_n = 0.0256 Re_n^{0.8} \frac{\theta^{0.25} A_n I^{0.25}}{x^{0.8} \left[ \int_0^x \theta^{1.25} x^{1.1} A_n I^{0.25} dx \right]^{0.2}}, \quad (15)$$

$$Nu = 0.0320 Re^{0.8} \frac{\left[ \int_0^x \theta^{1.25} x^{1.1} A_n I^{0.25} dx \right]^{0.8}}{X^{0.8} \int_0^x \theta x dx} \quad (16)$$

Here  $t_d(r) - t_0 = 0$ ,  $I(K_0, x) = \int_0^x x^{2.5} A_n dx$ . From Formulas (15) and

(16), for a constant disk temperature ( $\theta = \text{const}$ ), we find:

$$Nu_n = 0.0256 Re_n^{0.6} B_n(K_v, x),$$

$$B_n(K_v, x) = \frac{A_n /^{0.25}}{x^{0.5} \left[ \int_0^x x^{1.1} A_n /^{0.25} dx \right]^{0.2}}, \quad (17)$$

$$Nu = 0.0640 Re^{0.6} B(K_v, X), \quad B(K_v, X) = \frac{\left[ \int_0^X x^{1.1} A_n /^{0.25} dx \right]^{0.8}}{X^{0.5} (X^2 - 1)}, \quad (18)$$

The functions  $a(K_v, x)$ ,  $l(K_v, x)$ ,  $A_n(K_v, x)$ ,  $A(K_v, X)$ ,  $B_n(K_v, x)$  and  $B(K_v, X)$  for determining the flow stress, drag torque coefficient, and local and average heat transfer coefficients are calculated on an electronic digital computer. A portion of the results is presented graphically in Figures 1 — 5.

The dimensionless formulas (12), (14) — (18) are valid for  $Pr = 1$ . If the relationship (9) is used for the flow stress, then the calculation of the heat transfer gives  $Nu_n \sim Pr^{0.6}$ , based on the three-layer picture of the boundary layer according to the method of T. Karman in the limits  $5 \cdot 10^5 < Re_n < 10^7$ ,  $2 < x < 4$ ,  $1 < K_v < 10$ ,  $0.5 < Pr < 5$ . The same results are obtained in [7] for a freely rotating disk. Thus, in the first approximation, the effect of the Prandtl number can be taken into account by introducing the factor  $Pr^{0.6}$  into the formulas.

We compare the obtained approximate solutions for the hydrodynamic drag and heat transfer of the disk with experimental data.

Increase of the drag torque of a disk with flow was investigated by V. S. Sedach [1] (his experimental data were obtained for  $Re = 10^6$ ,  $\frac{s}{R} = 0.1$ ,  $X = 4$ ). The calculated and experimental values



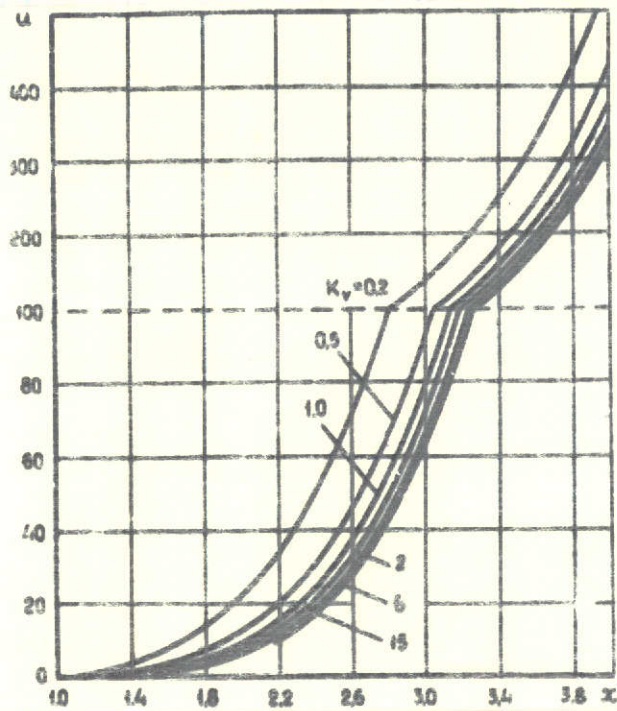


Figure 1. Values of the function  $a(K_v, x)$ .

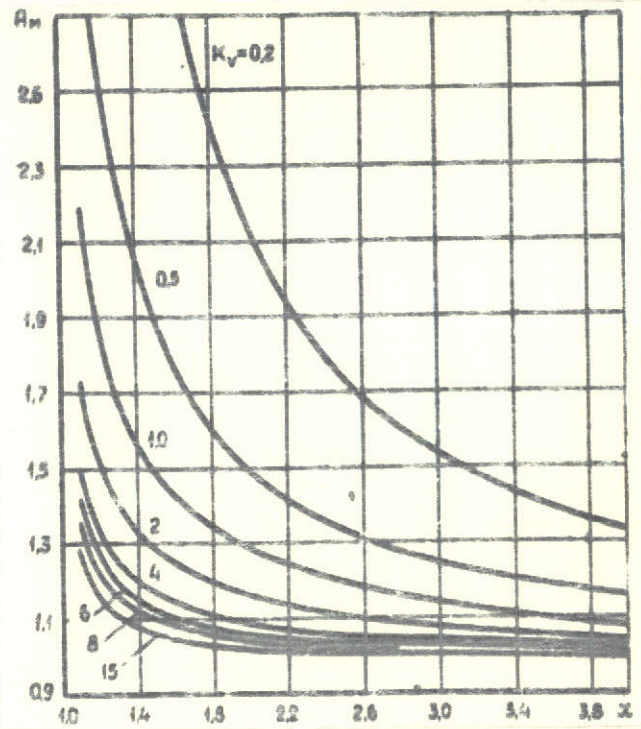


Figure 2. Values of the function  $A_m(K_v, x)$ .

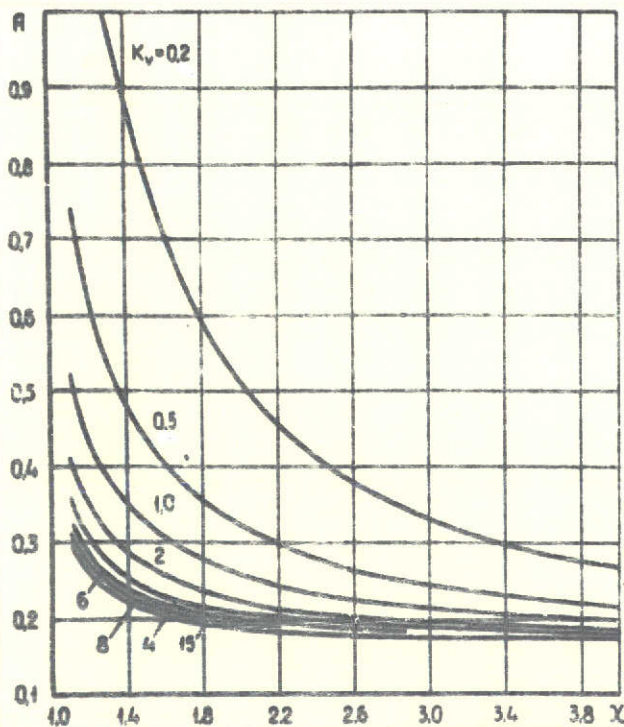


Figure 3. Values of the function  $A(K_v, x)$ .

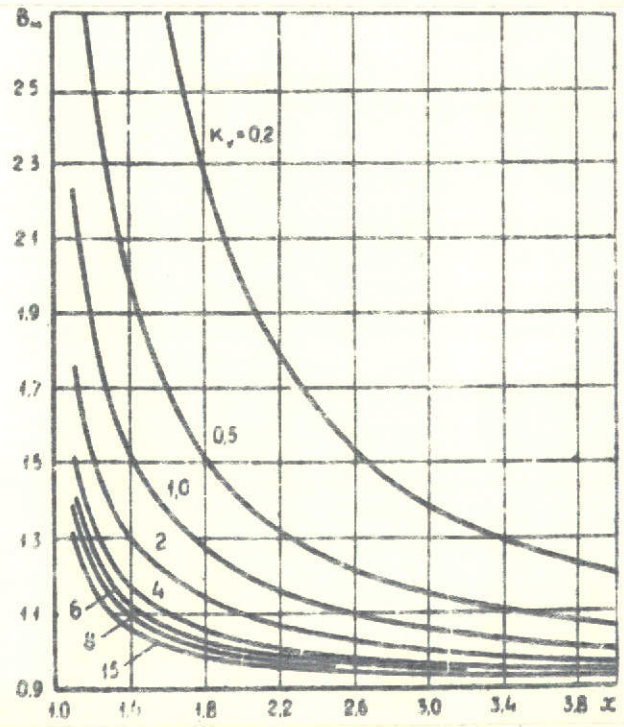


Figure 4. Values of the function  $B_m(K_v, x)$ .

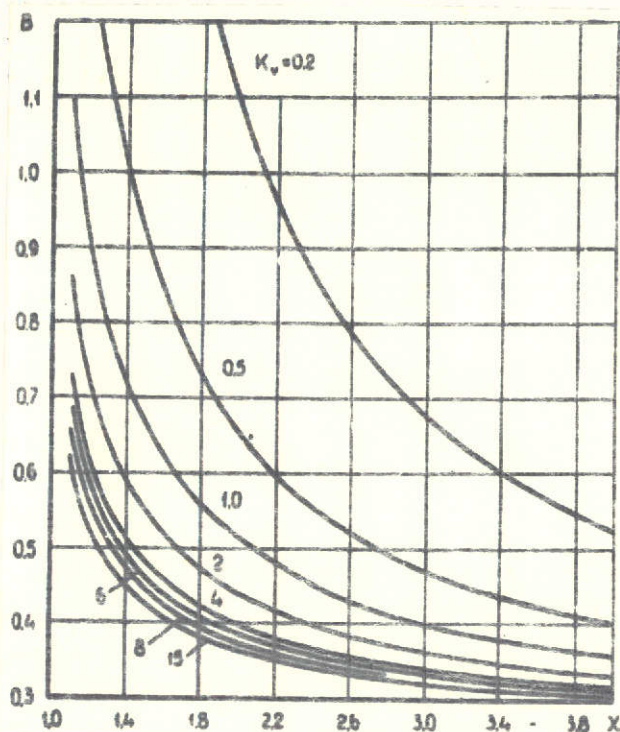


Figure 5. Values of the function  $B(K_v, X)$ .

$\Delta M$  N-cm

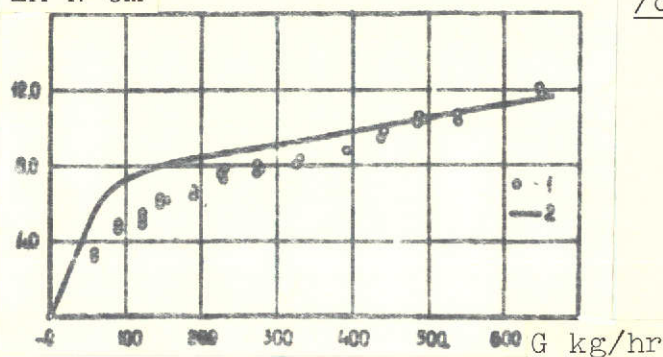


Figure 6. Increase of drag torque with flow rate in the gap between disk and casing. 1 — experimental data of V. S. Sedach; 2 — calculation.

are presented in Figure 6. The calculated values of drag torque are determined by summing over the section from  $x = 1$  to  $x_1$ , where the flow core exists between the boundary layers, and over the section from  $x_1$  to  $x$ , where the viscous disturbance region becomes equal to the gap width. Over the first section the torque is determined from Formula (10). To find the torque over the second section, the solution of L. A. Dorfman [9] was used, which is obtained under the approximation that the boundary layers are joined beginning at the input section. The coordinate  $x_1$ , where the joining of the boundary layers occurs, is located by successive approximations by summing the thicknesses of the boundary layers of the disk and casing according to the given value of  $K_v$ . The thickness of the boundary layer at the disk is calculated from Formula (7); at the casing — from the relationship:



$$\frac{\delta_0}{r} = 0.285 \frac{K_v^{0.2} (x^{1.25} - 1)^{0.8}}{Re_0^{0.2}}, \quad Re_0 = \frac{\omega r_0^2}{\nu}, \quad (19)$$

resulting from the momentum equation for the radial direction. The radial velocity distribution in the boundary layer at the casing is taken in the form  $V_r = V_0 \left( \frac{r}{\delta_0} \right)^{1/7}$

As is seen in Figure 6, the calculated and experimental values of  $\Delta M$  are in satisfactory agreement in the region of high flow rates, where the experimental determination of the torque can be assumed to be more accurate.

Results of an experimental study of heat transfer for a disk rotating in a thermally insulated casing with radial blowing are presented in [10]. The experimental data for a disk of thickness 45 mm in which the temperature distribution has a quadratic form are generalized by the dimensionless formula ( $Pr = 0.72$ ):

$$Nu = 0.0346 Re^{0.8} K_v^{-0.1} X^{-0.3} \left( \frac{s}{R} \right)^{0.05}, \quad (20)$$

In the experiments, the independent variables were varied in the limits  $5 \cdot 10^5 < Re < 4 \cdot 10^6$ ,  $0.6 < K_v < 7.0$ ,  $2.15 < X < 2.7$ . The dependence of the number  $Nu$  on  $X$  was obtained through two points:  $X = 2.15$  and  $X = 2.7$  for  $K_v = 2 = \text{idem}$ .

Approximation of the function  $A(K_v, x)$  in the interval  $2 < X < 3$  for  $K_v = 2$  (Figure 3) gives a relation coinciding with the experimental data with great accuracy:

$$A(K_v, X) = C_1 X^{-0.3}, \quad (21)$$



Approximating the function  $A(K_v, X)$  in the parameter  $K_v$  for  $X = 2.7 = \text{idem}$  (the dependence of  $Nu$  on  $K_v$  was established in the experiments in an analogous manner), we find:

$$A(K_v, X) = C_2 K_v^{-0.1}. \quad (22)$$

The coefficients  $C_1$  and  $C_2$  are averaged in such a manner that (21) and (22) give the same value for the function  $A(K_v, X)$  at the point  $X = 2.7$ ,  $K_v = 2$ . Then:

$$A(K_v, X) = 0.294 K_v^{-0.1} X^{-0.3}. \quad (23)$$

In the region bounded by  $2 < X < 3$ ,  $1 < K_v < 8$ , the average error in approximating the function  $A(K_v, X)$  by the power Formula (23) is less than 3.5%.

Substituting the approximate expression for the function  $A(K_v, X)$  into Equation (14) and introducing the factor  $Pr^{0.6} = 0.82$ , we obtain:

$$Nu = 0.0323 Re^{0.8} K_v^{-0.1} X^{-0.3}. \quad (24)$$

The calculated relation (24) assumes the existence of a nonviscous flow core in the gap between the disk and casing. In order that this condition be satisfied for the experimental relation (20), we determine that value of  $s/R$  for which the boundary layers are not joined for the whole investigated range of values of  $Re$ ,  $K_v$ ,  $X$ . The maximum thicknesses of the boundary layers at the disk and casing for  $Re = 5 \cdot 10^5$ ,  $K_v = 7$ ,  $X = 2.7$  give  $s/R = 0.125$ . It then follows from Formula (20) that:

$$Nu = 0.0306 Re^{0.8} K_v^{-0.1} X^{-0.3}. \quad (25)$$

The discrepancy between the calculated and experimental relations (24) and (25) with consideration of the error in approximating the function  $A(K_v, X)$  does not exceed 9%.

We compare Equation (17) for  $0 = \text{const}$  with the experimental data of A. L. Kuznetsov [5], which were obtained with uniform heating of the disk. The experimental relation for the local Nusselt number presented in [5] has the form in our notation:

$$Nu_M = 0.0235 (1 + b^2)^{3/8} (1 - \zeta)^{0.75} Re_M^{0.8}, \quad b = 0.25 + \frac{1}{XK_v x^2 (1 - \zeta)}. \quad (26)$$

The formula is valid for  $X = 2.7$ .

For small  $K_v$  the flow twist  $\zeta$  can be set equal to zero. Then, if  $K_v = 1$ ,  $X = x = 2.7$ , we obtain from Formula (26):

$$Nu_M = 0.0244 Re_M^{0.8}.$$

We note that in these experiments the calorimeters on the disk occupied a region bounded by the radii 135 and 278 mm; thus, the actual value  $x = 2$ . Setting  $x = 2$ ,  $K_v = 1$  in Equation (17), we find:

$$Nu_M = 0.0256 Re_M^{0.8} B_M(K_v, x) Pr^{0.6} = 0.0253 Re_M^{0.8}.$$

Agreement between the calculated and experimental relations also appears satisfactory in this case.

The experimental formula of E. Patrick and R. Smith is presented in [11], which can be represented in the form:

$$Nu_M = 0.123 (1 - \zeta)^{0.75} Re_M^{0.7} Pr^{0.4} \left( \frac{s}{r} \right)^{0.3} \frac{x^{0.6}}{(x^2 - 1)^{0.3}}. \quad (27)$$

Although it was noted in [11] that the scatter of experimental data is very large and thus great accuracy should not be expected from the formula, nonetheless, it is of interest to compare it with the approximate theoretical solution.

According to the formula of E. Patrick and R. Smith  $Nu_M = 1440 - 1770$  for  $Re = 10^6$ ,  $x = 3$ ,  $s/r = 0.05 - 0.1$ , and  $\zeta = 0$ . According to Formula (17), we have for  $K_v = 0.5 - 1.0$ ,  $Nu_M = 1440 - 1840$  for  $K_v = 3 - 6$   $Nu_M = 1270 - 1300$ . The numbers  $Nu_M$  appear close for small values of  $K_v$ ; with increasing  $K_v$  to  $3 - 6$ , it is necessary to take  $\zeta > 0$  in Formula (27).

We consider the question of the effect of the relative gap  $s/R$  on the heat transfer coefficient. It is obvious that this effect must appear, if the boundary layers of the disk and casing are joined in the peripheral part of the gap. Here one can use the method used to determine the drag torque: in the input section from  $x = 1$  to  $x_1$  the heat transfer coefficient is calculated from the formulas of the present work, in the section from  $x_1$  to  $X$  — from formulas obtained under the assumption that the boundary layers are joined [9]. As an example, results are tabulated from a determination of the average Nusselt number  $Nu_{av} = (\alpha_{av} R)/\lambda$  at the disk surface with a quadratic temperature distribution for various relative gaps  $s/R$ . In the calculations  $Re = 10^6$ ,  $X = 2.7$ ,  $K_v = 2.0$ . The  $Nu$  numbers of the two sections are averaged with the formula:

$$Nu_{av} = \frac{0.5Nu_1X(x_1^4 - 1) + 2Nu_2x_1 \int_{x_1}^x x^3(1 - \zeta) dx}{x_1 \left[ 0.5(x_1^4 - 1) + 2 \int_{x_1}^x x^3(1 - \zeta) dx \right]}; \quad Nu_1 = \frac{2_1 r_1}{\lambda}, \quad Nu_2 = \frac{a_2 R}{\lambda}.$$



The latter takes into account a quadratic variation of temperature with radius of the disk and cooling fluid in the second section.

As is seen in the table,  $Nu_{av}$  displays an extremum over a wide range of  $s/R$ . Initially, the heat transfer coefficient decreases with decreasing  $s/R$ ; then with the reduction of the dispersal section the heat transfer coefficient begins to increase. In general, the variation of  $Nu_{av}$  is small — about 5%. If  $\frac{s}{\delta_0 + \delta_R} > 0.5$ , then the effect of  $s/R$  on the heat transfer can be taken into account approximately, as in the experimental relation (20), by introducing the factor  $\left(\frac{s}{\delta_0 + \delta_R}\right)^{0.06}$ . The corresponding values of  $Nu^*$  are in the sixth column of the table. Values of  $Nu_c$ , calculated under the assumption that the boundary layers are joined beginning at the input section, are presented in the last column for comparison. As was to be expected, there is satisfactory agreement with the previous calculations in this case for small  $s/R$ .

TABLE\*\*

$\frac{s}{R}$	$x_1$	$Nu_1$	$Nu_2$	$Nu_{av}$	$Nu^*$	$Nu_c$
1	2	3	4	5	6	7
0.20	$>2.7$	1700	—	1700	1700	1090
0.10	$>2.7$	1700	—	1700	1700	1250
0.08	2.58	1610	1550	1670	1690	1320
0.06	2.25	1350	1590	1605	1660	1400
0.04	1.89	1105	1630	1620	1625	1500
0.02	1.45	865	1690	1670	1560	1630

\*\*Translator's Note: Commas in the numbers indicate decimal points.

In conclusion, we note the effect of the radial temperature gradient in the disk on the heat transfer rate.

The calculations show that the magnitudes of  $Nu_M$  and  $Nu$  determined from Formulas (12), (14), and (17), (18) — i.e., with quadratic and uniform temperature distributions with radius — differ within 10 — 15%. Thus, in many cases they can be applied directly. The accuracy of the formulas, as was shown in the above comparison with a number of particular empirical relations, is quite sufficient for engineering calculations over a wide range of  $Re$ ,  $K_v$  and  $X$ . Refinement of the heat transfer coefficients using Equations (15), (16) appears necessary only for thin disk rotors used in transport engines and under conditions of a nonstationary temperature field with large radial temperature gradients. The problem is solved by successive approximations. The possibility of constructing empirical formulas taking into account the effect of a radial temperature gradient is discussed in [6, 12].

#### REFERENCES

1. Sedach, V. S. Tr. KhPI (Trudy khar'kovskovo politekhnicheskovo instituta), Vol. XXIV, Issue 6, 1957.
2. Karman, T. ZAAM, No. 1, 1921.
3. Cobb, E. C. and O. A. Saunders. Proceedings of the Royal Society of London, No. 1206, 1956.
4. Daily, J. W. and R. E. Nece. Trans. of the ASME, Vol. 82, No. 1, 1960.
5. Kuznetsov, A. L. Author's abstract of dissertation, 1963.
6. Kapinos, V. M. IFZh (Inzhenerno-fizicheskiy zhurnal), No. 1, 1964.
7. Dorfman, L. A. Gidrodinamicheskoye soprotivleniye i teplootdacha vrashchayushchikhsya tel (Hydrodynamic Drag and Heat Transfer of Rotating Bodies), Moscow, State Physics and Mathematics Press, 1960.

8. Kapinos, V. M. Izvestiya AN SSSR, Energetiki i transport, No. 4, 1964.
9. Dorfman, L. A. Izvestiya AN SSSR, Mekhanika i mashinostroyeniye, No. 4, 1961.
10. Kapinos, V. M. Tr. KhPI, Vol. XXIV, Issue 6, 1957.
11. Traupel', V. Teploviye turbomashiny (Heat Turbines). State Power Engineering Press, 1963.
12. Kapinos, V. M. IFZh, No. 3, Iss. "Nauka i tekhnika," 1963.

Translated for National Aeronautics and Space Administration under contract No. NASw 2483, by SCITRAN, P. O. Box 5456, Santa Barbara, California 93108.